INTERACTION OF LIGHT WITH A TURBULENT LIQUID FLOW

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UDC 532.542.4.001.5:535.36

The relation between the rms fluctuation of a light current and the density fluctuation in a turbulent liquid flow is considered. Criteria are established for the effect of medium turbulence on the refractive index and on light attenuation. Experimental apparatus and new experimental data are described.

There exists a relatively small amount of data in the literature on interaction of light with a turbulent atmosphere [1, 2] and on light transmission through a turbulent liquid [3, 4].

The fluctuations of flow velocity u' in a turbulent current create, in turn, pressure, temperature, and impurity concentration fluctuations.

In this paper we restrict the consideration to pure media, whose turbulence is characterized by fluctuations of flow velocity and temperature only. The orders of pressure P' and temperature T' fluctuations are then determined by the equations

$$P' \approx \langle\rho\rangle \langle u\rangle u', \quad T' \approx Lu' \langle u\rangle^{-1} \mathrm{grad} \langle T\rangle \tag{0.1}$$

Here $\langle\rho\rangle$ is the average liquid density, $\langle u\rangle$ is the average flow velocity, L is a characteristic length, and $\langle T\rangle$ is the average temperature.

For density fluctuations of the medium we have, correspondingly,

$$\rho' \approx \frac{\partial\rho}{\partial T} T' + \frac{\partial\rho}{\partial p} P' \qquad (\rho' \ll \langle\rho\rangle) \tag{0.2}$$

where $\langle p\rangle$ is the average pressure.

The quantity $\partial\rho/\partial p$ is inversely proportional to the square of the sound propagation velocity in the given medium.

1. Fluctuation of the Refractive Index of Light

The refractive index n is related to the medium density ρ by the Lorentz–Lorenz equation [5]

$$(n^2 - 1)/(n^2 + 1) = C\rho \tag{1.1}$$

where C is constant for the given material.

Applying to this function the well-known Reynolds rule [6], we obtain the following relation between the fluctuation in the light refractive index n' and the fluctuation in the medium density

$$3 \langle\rho\rangle (\langle n\rangle^2 - 1)^{-1} (n'n' + 2 \langle n\rangle n') - \langle n\rangle^2\rho' - 2 \langle n\rangle n'\rho - n'n'\rho' = 0 \tag{1.2}$$

Hence we have for the averaged fluctuations

Tibilisi and Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 115-123, July-August, 1973. Original article submitted May 29, 1972.

$$3\langle\rho\rangle(\langle n\rangle^2-1)^{-1}\langle n'n'\rangle+2\langle n\rangle\langle n'\rho'\rangle+\langle n'n'\rho'\rangle \qquad (1.3)$$

Equation (1.3) connects the rms fluctuation of the refractive index with the fluctuation correlations of the refractive index and of the medium density. Generally (for a strongly nonisothermic gas flow, or a liquid flow at nearly critical thermodynamic parameters) the fluctuation may be comparable to the average density. In an isothermic or quasiisothermic liquid flow, however, $\rho' \ll \langle\rho\rangle$, and, correspondingly, $n' \ll \langle n\rangle$. In this case

$$n' \approx (n^2-1)(n^2+2)\rho'/6n\langle\rho\rangle \qquad (1.4)$$

Substituting (1.4) in the first (main) term of the right-hand side of (1.3), we have

$$\langle n'n'\rangle \approx [(n^2-1)/3\langle\rho\rangle]^2(n^2+2)\langle\rho'\rho'\rangle \qquad (1.5)$$

It should be kept in mind that the averaging operation is performed over the correlation radius, since the time it takes light to traverse a distance equal to a characteristic turbulence scale is much shorter than the time scale of existence of turbulent perturbations.

It is seen from (1.5) that the rms fluctuation of the refractive index of light in a turbulent medium is nonvanishing. This implies that the effective refractive index in a turbulent medium is larger than that determined by its physical properties.

Density fluctuations due to temperature variation are usually important in gases, particularly in light transmission through the atmosphere of the earth.

For pure media, particularly for light transmission in the ocean, isothermic conditions may arise, and turbulence will appear through the action of pressure fluctuations, as if the latter are small.

For $T=$ const, $T'=0$, and it follows from (0.1), (0.2), and (1.5), that

$$(\langle n'n'\rangle)^{1/2} \approx (n^2-1)\,\psi M^2, \quad \psi \approx (\langle u'u'\rangle)^{1/2}/\langle u\rangle \qquad (1.6)$$

where ψ is the degree of flow turbulence, and M is the Mach number.

In first approximation, obviously, it may be assumed that the apparent enhancement of the light refractive index in a turbulent liquid Δn_T is of the order of the rms fluctuation of the physical refractive index, i.e.,

$$\Delta n_T \approx (\langle n'n'\rangle)^{1/2} \qquad (1.7)$$

To determine the variation of the refractive index, related to the inhomogeneous turbulence, the experimental apparatus shown schematically in Fig. 1 was mounted.

The basic unit is an ITR-1 interferometer, whose principle of action is based on diffraction from a double slit.

A two-chamber cell 3,4 is located on the path between the diaphragm 1 and the object 2. The cells are filled by the investigated liquid; cell 4 has a sample and the liquid in cell 3 is in motion.

The difference of refractive indices of the resting and moving liquids creates a phase shift between the rays passing through the different cells, which causes a displacement of the interference pattern observed in ocular 5.

Doubly distilled water, supplied to the cells by a pump 6 from a reservoir 7, was used in the studies. Before the final filling the apparatus was multiply flushed by alcohol, and then by doubly distilled water.

In performing experiments much attention was paid to careful thermostatic stabilization of the investigated liquid, as a temperature variation generated a variation in the refractive index of the liquid.

The liquid in the reservoir 7 was thermostatically stabilized by a coil 8, inside which the liquid was circulated from a thermostat 9. A UT-15 ultrathermostat, with special thermal isolation, served as reservoir. The circulating pump 6 was submerged in water all the way up to the electric motor, with the purpose of preventing suction of air bubbles through the pump box. The cells 3, 4 were connected to the

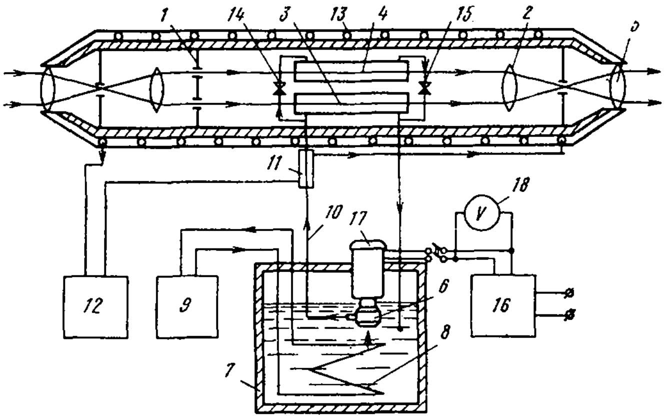

Fig. 1

pump by a copper tube 10 of diameter 12/14 mm. The tube 10 passes in the cell through a heat exchanger 11 of the "tube inside tube" type. The thermostatically controlled liquid, fed by the thermostat 12, was circulated in the external tube of the heat exchanger. Such a scheme guarantees constant temperature of the investigated liquid. For smaller effects of temperature fluctuation in the surrounding medium the room was especially thermostatically controlled. Besides, a copper wire heat exchanger 13 with a thermally isolating outer coating was mounted on the interferometer. Water from the thermostat 12 was fed into heat exchanger 13, passing through heat exchanger 11.

The temperature in thermostats 9 and 12 was maintained identical within 0.01°, and at 2-3° above room temperature.

The tube 10, through which the investigated liquid passed, had a tap for water transmission into cell 4. Thus, the liquid in both cells had identical temperature. Before each experiment the water in cell 4 was changed by means of valves 14 and 15. Water flow in the cell was regulated by an RNO transformer 16, supplied by the pump electric motor 17.

The feeding current was controlled by a voltmeter 18 of class 0.2. The water flow was determined by the voltage of the current supplied by the pump electric motor 6.

All experiments were performed at a liquid temperature of 293°K, with cells of length 250 and 500 mm.

The instrument output in its operating region was controlled by the stability of absence of a phase shift between the rays passing through the cell for the resting liquid.

At fixed water flow the path difference of rays passing through layers of resting and moving liquid was determined, after which the variation in the refractive index of distilled water, Δn_T, generated by motion, was determined by a standard calculation. Measurement results, performed on a cell of length 250 mm and diameter 10 mm, are shown in Fig. 2 as a function of $\Delta n_T = f(K^*)$.

Here

$$K^* = (n^2 - 1) M^2 [(n^2 + 2)^{1/2} C_f]^{1/2}$$

where C_f is the friction coefficient.

Those results are in qualitative and order-of-magnitude agreement with the theoretical estimates given above.

2. Coefficient of Attenuated Ray

Figure 3 shows the dependence of the attenuated laser beam intensity on the liquid flow velocity and viscosity. The liquids investigated were distilled water, freon-113, and the polymethylsiloxane liquids PMS-1.5 and PMS-50.

The experimental data shown in Fig. 3 were obtained for the following tube sizes and liquid parameters.

Distilled water: a) the points 1, 2, 3 correspond to experiments in tubes of diameter d = 50 mm and

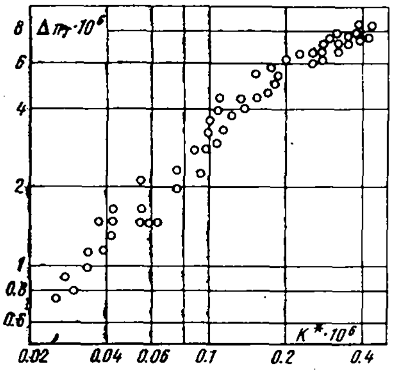

Fig. 2

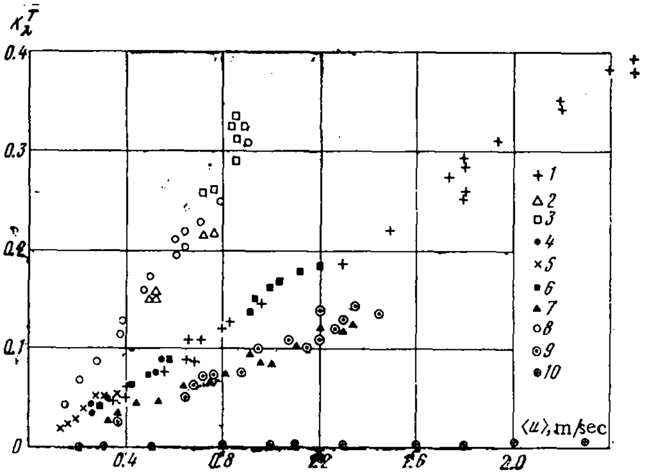

Fig. 3

length $l = 2.65$ m at the following viscosities: $\nu = 1 \cdot 10^{-6}$, $0.44 \cdot 10^{-6}$, and $0.42 \cdot 10^{-6}$, respectively; b) the points 4 correspond to d = 100 mm, $l = 2.65$ m, and $\nu = 1 \cdot 10^{-6}$; c) the points 5 correspond to d = 140 mm, $l = 2.65$ m, and $\nu = 1 \cdot 10^{-6}$; d) the points 6 correspond to d = 72 mm, $l = 1.4$ m, and $\nu = 1 \cdot 10^{-6}$.

Ethyl alcohol: the points 7 correspond to d = 72 mm, $l = 1.4$ m, and $\nu = 1.5 \cdot 10^{-6}$.

Freon-113: the points 8 correspond to d = 50 mm, $l = 2.65$ m, and $\nu = 0.45 \cdot 10^{-6}$.

PMS-1.5: the points 9 correspond to d = 72 mm, $l = 1.4$ m, and $\nu = 1.5 \cdot 10^{-6}$.

PMS-50: the points 10 correspond to d = 28 mm, $l = 1$ m, and $\nu = 50 \cdot 10^{-6}$.

As is seen, in laminar flow (high viscosity) the effect of motion is practically not apparent on the attenuated light beam. For turbulent flow this effect is important.

Light scattering by turbulent fluctuations causes light attenuation in addition to that due to molecular mechanisms. Assuming simple summation of these effects, we write down Buerger's equation for the case considered in the form

$$dI = -(K_\lambda + K_\lambda^T)\, I dx \tag{2.1}$$

Here K_λ is the attenuation coefficient, K_λ^T is the turbulent attenuation coefficient, and I is the radiation intensity.

The attenuation (scattering) coefficient by the turbulent fluctuations can be assumed to be some function of Δn_T and of the degree of turbulence (i.e., the characteristic Reynolds number of the flow), and is inversely proportional to the number of collisions with small-scale turbulent formations $\Lambda_{\min}$, inside which light interacts with matter on a molecular level only.

Assuming

$$\Lambda_{\min} \approx \nu\, (\langle u'u' \rangle)^{-1/2} \tag{2.2}$$

we have, accurately up to the degree of turbulence, the criterion [7]

$$\tilde{K}_\lambda^T = \nu K_\lambda^T / \langle u \rangle \tag{2.3}$$

so that

$$K_\lambda^T = f\,(\mathrm{Re},\ \Delta n_T) \tag{2.4}$$

Experiments in determining the quantity K_λ^T were performed on the apparatus schematically represented in Fig. 4. Axial flow in a circular tube was chosen as an object with uniform turbulence. At the end of tube 1 we have a box 2, optically attached to a quartz glass 3. Teflon gaskets 4 serve for consolidation.

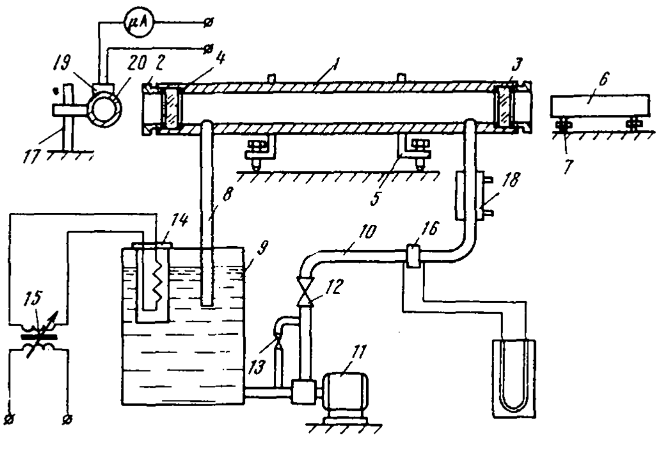

Fig. 4

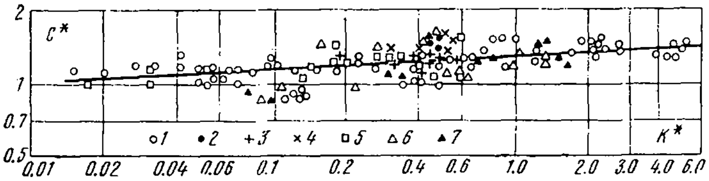

Fig. 5

Connecting pieces 8 and 10 were used for feeding and removing liquid in the experimental channel. The whole system, including the circulating pump 11, is made of stainless steel. A cooler 18 served to remove heat generated by hydrodynamic friction. A tank 9 of capacity ~100 liters was equipped by an electric heater 14, supplied by a regulating transformer 15. The flow of the circulating liquid is measured by a sharp disk 16. The flow smoothness is guaranteed by by-pass lines and valves 12 and 13. Adjustment of the experimental channel takes place by a support 5. A laser 6, attached to the support by adjustable screws 7, served as light source.

The laser beam was aligned along the axis of the experimental channel. An aperture in a Dural sphere 20 whose interval surface was coated by a barium oxide layer with a large reflection coefficient served as intake. The aperture diameter is 25 mm, and the sphere diameter is 115 mm. The stand 17 allowed displacement of the photodetector in vertical and horizontal directions. A photoelement 19 of FSK-1 type, supplied by a current regulator, was mounted to the top portion of the sphere. The experiments were performed with constant control of isothermic liquid flow. The light intensity passing through the resting liquid I_1 was first determined, and then the intensity I_2 for a given flow velocity.

The attenuation coefficient of light by turbulent fluctuations is

$$K_\lambda{}^T = -\, l^{-1} \ln I_2 / I_1 \qquad (2.5)$$

where l is the path length of light in the experimental setup.

As already mentioned, the liquid motion strongly affects attenuation of light passing through while disturbing the laminar flow. Significant fluctuations in I_2/I_1 were observed in the transition region of Reynolds numbers, caused by an intermittence effect. The light attenuation became stabilized in the region of turbulence development.

The Reynolds number

$$\mathrm{Re} = \langle u \rangle\, d / \nu \qquad (2.6)$$

varied to variations in the average flow velocity of the liquid $\langle u \rangle$, in the tube diameter d, and in the kinematic viscosity ν. For a given liquid the latter varied due to temperature enhancement by the heater 14.

Within the region of hydrodynamic parameters investigated no effect of light wavelength was observed

TABLE 1. Sizes of Experimental Components

Stainless steel tubes		Brass tubes	
d, mm	l, mm	d, mm	l, mm
20	1000, 2650	26	1000
28	1000	72	1400
38	1000		
50	1000, 2650		
100	2650		
140	2650		

TABLE 2. Characteristics of Lasers Used in Experiments Described

Type	Gas	Wavelength, μ	Power, MW	Beam diameter, mm
ЛГ-106	argon	0.51	1000	3.0
ЛГ-75	helium-neon	0.63	20	4.0
ЛГ-56	helium-neon	0.63	2	2.5
ОКГ-12	helium-neon	0.63	10	10.0
ЛГ-36	helium-neon	0.63	40	4.0

from 0.51 to 0.63 μm.

Figure 5 shows experimental data on light beam attenuation while passing along the flow axis in tubes of diameter larger than 50 mm, in coordinates $C^* = f(K^*)$ corresponding to Eq. (2.4). Here

$$C^* = \frac{\nu K_\lambda{}^T}{\langle u \rangle} \left(\frac{2}{C_f} \right)^{1/2}$$

The liquids investigated were distilled water [points 1) $\nu = 1 \cdot 10^{-6}$, 2) $\nu = 0.8 \cdot 10^{-6}$, 3) $\nu = 0.44 \cdot 10^{-6}$; 4) $\nu = 0.42 \cdot 10^{-6}$], ethyl alcohol [points 6) $\nu = 1.5 \cdot 10^{-6}$], freon-113 [points 5) $\nu = 0.45 \cdot 10^{-6}$)], and PMS-1.5 [points 7) $\nu = 1.5 \cdot 10^{-6}$].

The experimental points can be described by a straight line in the range of parameters investigated. Other conditions being equal, an appreciable decrease in K_λ^T is noticed in tubes of diameter less than 50 mm. The nature of this variation is shown below.

d (mm), $\langle u \rangle$ (m/sec)	20 8.5	26 5	28 5	38 3.2	⩾50 0.8	⩾50 0.4
K_λ^T	0.02	0.045	0.065	0.085	0.12	0.065

The characteristics of the tubes and lasers used are shown in Tables 1 and 2.

3. Fluctuations in Light Intensity

An FÉU-22 light detector, placed in a special metallic body so as to reduce external interference, was used to measure light intensity fluctuations in the instrument (Fig. 4).

A light-sensitive window, on which the light beam is incident, was blocked by a diaphragm with a scale division of 7 μ. The FÉU signal entered a C-1-15 oscilloscope.

The fluctuation curves observed on the oscilloscope screen were photographed and processed by the equation

$$\beta = [(\langle I \rangle - I)^2 \langle I \rangle^{-2}]^{1/2} \tag{3.1}$$

Knowing the magnitude of the light intensity fluctuation β, one can evaluate the dispersion of the logarithm of intensity fluctuation σ^2 by the following equation:

$$\sigma^2 = \ln(1 + \beta^2) \tag{3.2}$$

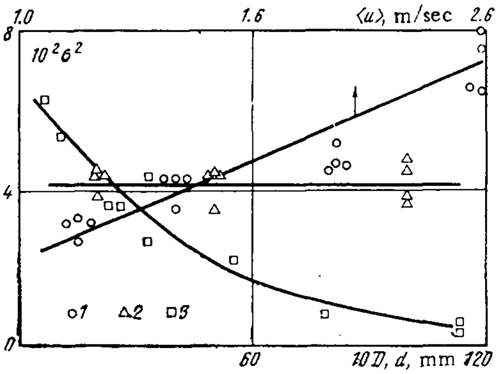

Fig. 6

TABLE 3

l, m	$M \cdot 10^4$							
	6.8	8.2	8.8	9.2	10.3	12.3	13	13.7
2	23	—	—	34	—	38	—	47
2.65	20	—	34	—	—	41	—	44
3.8	18	24	—	—	29	33	35	—

Results of several experiments, obtained for distilled water in tubes of d = 50 mm and l = 2.65 m, are shown in Fig. 6 (points 1). As is seen, the logarithm dispersion of the light intensity fluctuation increases with increasing flow velocity of the turbulent liquid.

No effect of the experimental tube diameter on the quality σ^2 was observed (see Fig. 6, points 2) for a receiving aperture diameter 3.3 mm and Re = $6.5 \cdot 10^4$. The observed effect depends, however, on the size of the receiving aperture. In the experiments the magnitude of the receiving aperture diameter D varied from 0 to 11.5 mm (see Fig. 6, points 3). The logarithmic dispersion of light intensity fluctuation drops with increasing diameter and then saturates around a receiving aperture diameter approximately equal to the diameter of the light beam leaving the experimental tube. The experimental data of Fig. 6 were obtained on distilled water.

Besides intensity attenuation and fluctuation, the light beam passing through the turbulent liquid undergoes further broadening.

Experimental measurements on an LG-75 laser beam broadening were performed on the instrument in Fig. 4. The light beam leaving the experimental tube was incident on a diaphragm of aperture 7 μm. The diaphragm could be moved in the horizontal plane perpendicular to the light beam axis and was rigidly attached to a photomultiplier. An FÉU-62 photomultiplier moved together with the diaphragm by means of a micrometric screw with scale division 10 μm. The FÉU signal passed through an amplifier on to an M-26 microammeter of type 0.5. The light beam boundaries could be sharply determined from the microammeter readings.

Table 3 shows the dependence of the laser beam broadening $\Delta R/l \cdot 10^6$ m on the Mach number (M) in the turbulent flow region for a series of experiments on distilled water at various tube lengths.

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